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Learning to Learn

The Art of Doing Science and Engineering

Session 11: Coding Theory II

Coding Theory II Overview

Two types of Coding

- Source Encoding
- Channel Encoding

How do we find the best source encoding?

- Huffman encoding

How do we encode for a noisy channel?

- Error correcting codes

The Best Source Encoding

We know efficient codes are related to probabilities.

Intuition leads to the solution:

$$p_1 \geq p_2 \geq p_3 \ldots \geq p_n$$
$$l_1 \leq l_2 \leq l_3 \ldots \leq l_n$$

Prove this by exchanging any two lengths and the average code length decreases.

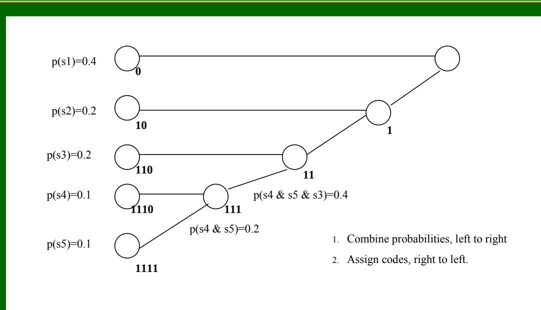
Deriving Huffman Encoding

Create the best encoding of symbols from their probabilities.

Build encoding tree from bottom up.

- Merge 2 least frequent nodes, combine their probabilities, repeat.
- Assign 0,1 to final 2 symbols.
- Back down the tree, split symbols, assigning 0 or 1 appended to parent's code.

Deriving Huffman Encoding Tree (Visual)



Optimality

Huffman trees give optimal (most efficient) codes.

Do we necessarily want optimal?

What characteristics do we want?

- Efficiency (average code length)?
- Efficiency and low variability?

Start from an optimum and move one way or the other.

- Sometimes you can get a large change a desired characteristic but pay little in performance.

Why Huffman?

Write a program to do it.

Sending Huffman tree and encoded info saves channel or storage capacity.

Best possible data compression.

- If anything can be gained by compression.
- With uniform distribution, thus block code, no compression possible.

Channel Encoding

What to do with noise?

- Error detection

Extra parity bit catches only 1 error.

Probability of 2 errors in a message of length *n* increases with *n*.

- In a longer message parity bit takes up less channel capacity.
- In a longer message 2nd error more likely.
- Depends on probability *p* of error in a single bit.

Errors

Probability of 2,3,4, and higher errors depends mostly on *np* product.

- *np* near 1 makes errors highly likely.
- Very small *np* makes double, triple errors highly unlikely.

Tradeoff between error and efficiency.

Engineering judgment that requires a concrete case and a real *n* and *p*.

Error Handling

What if an error occurs?

Repeat the message

- On marginal errors repeat messages will be clear
- On systematic errors you will repeat error until you get a double error. Not a good strategy.

Strategy depends on the nature of the errors you expect.

Detecting Human Error

Encoding alphanumeric symbols.

Humans make different types of errors than machines

- Changing one digit: 667 to 677
- Exchanging digits: 67 to 76
- Parity can't cope with the two digit exchange error

Detecting Human Errors

Weighted code

- Add 1 parity symbol to a message of length *n-1* so the message checksum below is 0.

x_k is the value for the k^{th} symbol of the message, $0 \le x_k \le 36$

$$\left[\sum_{k=1}^{n} k \cdot x_k\right] \bmod 37$$

Different Types of Codes

- ISBN is a weighted code with modulo 11.
- Even/odd parity bit is for white noise
- Weighted code is for human noise.
  - But machine can easily check code for mistake
- Many types of codes not talked about.
- Know what the noise is to determine type of code.
- Design a code to meet the cope with your particular type of noise problem.

Creativity in Deriving Codes

- Learn principles, then solve your problem.
- Next lecture will tell how Hamming derived his codes.
- If you were in the same situation Huffman or Hamming were in, could you come up with the codes if you had to?
- Creativity is simple if you are prepared.

Parting Thoughts

- Many scientists wasted the second half of their lives elaborating a famous theory.
  - Leave elaborations to others.
  - Follows from Hamming's earlier statement, that those who come up with great ideas often don't understand their own ideas as well as do those who follow.
- Set great work aside, try to do something else.