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Learning to Learn

The Art of Doing Science and Engineering

Session 17: Digital Filters IV

Recursive Filters

$$y_n = \sum_{j=0}^{K}[c_j u_{n-j}] + \sum_{j=1}^{K}[d_j y_{n-j}]$$

We have values on only one side of the current signal values u_n, and old values of the outputs, y_n.

If we did have "future values" then two sided prediction would probably be much more accurate.

Recursive Filters

The next thing we see is that the use of old output as new input means that we have feedback- and that automatically means questions of stability.

In a linear system we see that whatever pure frequency we put into the filter when it is steady state, only that frequency can emerge, though it may be phase shifted.

Recursive Filters

Transients can have other frequencies which arise from the solution of the homogeneous difference equation.

We are solving a difference equation with constant coefficients with the u_n terms forming the "forcing function"- that is exactly what a recursive filter is, and nothing else.

Recursive Filters

Therefore steady state is assumed. This ignores all transients.

$$x_n = A_I \exp\{iwt\}$$

$$y_n = A_O \exp\{iwt\}$$

$$A_O / A_I = \sum_{j=0}^{K} c_j \exp\{-ijw\} / [1 - \sum_{j=1}^{K} d_j \exp\{ijw\}]$$

Recursive Filters

This is a rational function in the complex variable exp{ijw} = z rather than, as before with non-recursive filters, a polynomial in z.

There is a theory of Fourier series representation of a function; there is not as yet a theory of the representation of a function as the ratio of two Fourier series.

Feedback

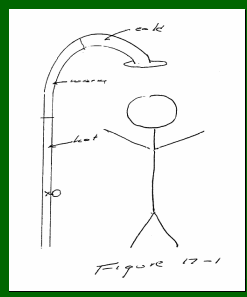
Design methods are not systematic.

Have Butterworth, two types of Chebyshev bands, and elliptic filters that have equal ripple in both.

Instability!! Developed a theory in coping with corrector formulas for numerically solving ordinary differential equations.

$$y_n + 1 = \sum_{j=0}^{K} [c_j y_{n-j}] + \sum_{j=0}^{K} [d_j y'_{n-j}]$$

Shower Pipeline



Recursive Filter

In standard non-recursive filters there is no feedback paths- the y_n that are computed do not appear later on the right hand side.

Recursive filters are often called “infinite impulse response filters” (IIR) because a single disturbance will echo around the feedback loop, which even if the filter is stable will die out only like a geometric progression.

Dealing with the Expert

Experts were told something in class when they were students first learning things, and at the time they did not question it.

What is learned becomes accepted fact, which they repeat and never really examine to see if what they are saying is true or not, especially in the current situation.

Bell Telephone Lab Story

Scientist was measuring the number of counts in a radioactive experiment at 256 energy level. He needed the derivative of the data.

Hamming sized up the scientist’s lab procedures to makes sure his physics was correct, and then connected him with Kaiser.

“To the extent that you can choose, then work on problems that you think will be important.”

Bell Telephone Lab Story

Problem solving with two experts in divergent fields has it problems. The curse of the expert is their limited view of what they can do. Outcome:

The program: 1) designed the corresponding differentiating filter, 2) wrote the program to compute the smoothed output, and 3) processed the data through this filter without any interference from the physicist.

Bell Telephone Lab Story

Once things start going right, there was a tendency to cut corners and cheat the integrity of the work they were doing.

Once convinced to get the most they could out of the expensive data, the scientist and Kaiser wrote a classic paper in the area, as it opened the door on a new range of things that could be done.

Contributions

First identify the problem.

Get the right people together, and mediate the experts lack of perspective in seeing different ways of looking at the problem.

There is an increasing need for people that have these skills, because people are getting more and more specialized and narrower and narrower in their knowledge.

Someone has to keep the larger view and see to it that things are done honestly.

Signal Processing

Most signal processing is indeed done on time signals. But most digital filters will probably be designed for small, special purpose studies, not necessarily signals in time.

You will need a digital filter to smooth the data to get a glimpse of the trend, if it exists.

Signal Processing

You don't want to find a trend when it does not exist, but if it does you want to know what it has been, so you can project what it is likely to be in the near future.

You will probably design many more filters for such odd jobs than you will for radar data reduction and standard things. It is usually in the new applications of knowledge where you can expect to find the greatest gains.

Misuse of Intellectual Tools

Fourier analysis implies linearity of the underlying model. You can use it on slightly nonlinear situations, but often elaborate Fourier analyses have failed because the underlying phenomena was too nonlinear.

Millions of dollars have been lost because people have persisted in doing the wrong thing.

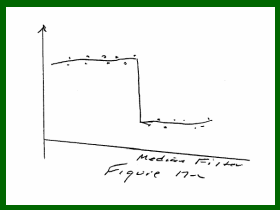
Running Median Filter

Computing the running median produces an output which smoothes out any local noise- the median will be near the average, which is a straight line least squares fit used for local smoothing.

Running Median Filter

At discontinuity we picture a flat level curve and then drop to another flat curve.

With an odd number of terms in the median filter, you see that the output will stay up until you have more than half of the points on the lower level, where upon it will jump to the lower level.



Running Median Filter

It will follow the discontinuity, and it will not try to smooth out the curve. In some situations that is the filtering you want. It removes the local noise but it does not lose the sudden changes in the state of the system being studied.

Final Observation on Digital Filters

There is a theorem which states that the variability of the function times the variability of its transform must exceed a certain constant. I said to myself, "What else is this than the famous uncertainty principle of Quantum Mechanics?"

Every linear theory must have an uncertainty principle involving conjugate variables. Once you have adopted the linear approach, and QM claims absolute additivity of the eigenstates, then you must find the uncertainty principle.

Final Life Observation as it Relates to Digital Filters

If you do not doubt accepted rules it is unlikely that you will be a leader into new areas; if you doubt too much you will paralyzed and will do nothing.

When to doubt, when to examine the basics, when to think for yourself, and when to accept things as they are, is a matter of style, and there is no simple formula on how to decide.

Final Life Observation as it Relates to Digital Filters

Bid advances usually come from significant changes in the underlying beliefs in the field.

As our state of knowledge advances the balances between aspects of doing research change.