THE ART OF DOING SCIENCE AND ENGINEERING

Learning to Learn

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PREFACE

After many years of pressure and encouragement from friends I decided to write up the graduate course in Engineering I teach at The Naval Postgraduate School. At first I began to concentrate on all the details which I thought should be tightened up rather than leave the material as a series of somewhat disconnected Lectures. In the past the Lectures often followed the interests of the students, and many of the later Lectures were suggested topics that they expressed an interest in. Furthermore, the Lectures changed from year to year as various areas developed. Since Engineering depends so heavily these days on the corresponding Sciences, I often use the words interchangeably.

But on more thought I decided that since I was trying to teach "style" of thinking in Science and Engineering, and that "style" is an art, I should therefore copy the methods of teaching used for the other arts (once the fundamentals have been learned). How to be a great painter cannot be said in words; one learns by trying many different approaches which seem to surround the subject. The usual art teacher lets the advanced student paint, and then makes suggestions on how they would have done it, or what might also be tried, more or less as the points arise in the student's head - which is where the learning is supposed to occur! In this series of Lectures I must somehow communicate to the students, what cannot be said in words - that is, the essence of style in Science and Engineering. Therefore I have adopted a loose organization with some repetition since this often occurs in the Lectures. There are, therefore, digressions and stories (with some told in two different places!), all in a somewhat rambling, informal style of typical Lectures.

I have also used the story approach, often emphasizing the initial part of the discovery, because I firmly believe in Pasteur's remark, "Luck favors the prepared mind." This way I can illustrate how the individual's preparation before they met the problem often led to the recognition, formulation, and solution of it. Great results in Science and Engineering are too often bunched in the same person for success to be a matter of ran-

dom luck.

<u>Teachers should prepare the student for the student's future, not for the teacher's past.</u> Most teachers rarely discuss the important topic of the future of their field, and when this is pointed out to them they usually reply with a remark like, "No one can know the future." It seems to me that the difficulty of knowing the future does not absolve the teacher from seriously trying to help the student to be ready for it when it comes. It is obvious that the experience of an individual is not necessarily that of a class of individuals, therefore any one person's projection into the future is apt to be somewhat personal and will not be universally accepted, but this does not justify going to impersonal surveys and losing the impact of the personal story.

Since the classes were almost all carefully selected Navy, Marine, Army, Air Force, and Coast Guard students, with a very few civilians, and, interestingly enough, about 15% very highly selected foreign military, the students face a highly technical future, hence the importance of preparing them for their future and not just our past.

The year 2020 is a convenient date to center the preparation for their future - sort of 2020 foresight, as it were. Being graduate students, mainly getting a Masters Degree at the end of the current term, they have the basics well in hand, and that leaves me the task of adding "style" to their education, which in practice is usually the difference between an average person and a great one. The school has allowed me great latitude in trying to teach a completely non-technical course; if you wish to say it this way, this course complements the more technical courses. As a result my opening words, occasionally repeated during the course, are: "There is really no technical content in the course, though I will, of course, refer to a great deal of it, and hopefully it will generally be a good review of the fundamentals of what you have learned. Do not think that the apparent content is the course - it is only illustrative material. Style of thinking is the center of the course."

The sub title, "Learning to Learn", is the main solution I have to offer to help the student cope with the rapid changes in their fields that they will have to endure, hence the course centers around how to look at, and think about, knowledge, and it supplies some historical perspectives that might be useful.

The course is mainly personal experiences I have had and digested, at least to some extent. Naturally one tends to remember one's successes and forget the lesser events, but I tell a number of my spectacular failures as clear examples of what to avoid. I have found that to be effective the personal story is far, far more effective than the impersonal one; hence there is necessarily an aura of "bragging" in the book that is unavoidable.

Let me repeat what I earlier indicated. Apparently an art,

which almost by definition cannot be put into words, is probably best communicated by approaching it from many sides, and doing this repeatedly hoping thereby that the students will finally master enough of the art, or if you wish, style, to significantly increase their future contributions to society. A totally different description of the course is that it covers all kinds of things that could not find their proper place in the standard curriculum.

The casual reader should not be put off by the mathematics; it is only "window dressing" used to illustrate and connect up with earlier learned material. Usually the underlying ideas can be grasped from the words alone.

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LECTURE 1

ORIENTATION

The purpose of this course is to prepare you for your technical future. There is really no technical content in the course, though I will, of course, refer to a great deal of it, and hopefully it will generally be a good review of the fundamentals you have learned. Do not think that the apparent content is the course - it is only illustrative material. Style of thinking is the center of the course.

I will examine, criticize, and display styles of thinking. To illustrate the points of style I will often use technical knowledge that most of you know, but, again, it will be, I hope, in the form of a useful review that concentrates on the fundamentals. You should regard this as a course that complements the many technical courses you have learned. Many of the things I will talk about are things that I believe you ought to know but which simply do not fit into courses in the standard curriculum. The course exists because the department of Electrical and Computer Engineering of the Naval Postgraduate School recognizes the need for both a general education and the specialized technical training that your future demands.

The course is concerned with "style", and almost by definition style cannot be taught in the normal manner by using words. I can only approach the topic through particular examples, which I hope are well within your grasp, though the examples come mainly from my 30 years in the mathematics department of the Research Division of Bell Telephone Laboratories, (before it was broken up). It also comes from years of study of the work of others.

The belief that anything can be "talked about" in words was certainly a belief of the early Greek philosophers, Socrates (469-399), Plato (427-347, and Aristotle (384-322). This attitude ignored the current <u>mystery cults</u> of the time who asserted that you had to "experience" some things which could not be communicated in words. Examples might be the gods, truth, justice, the arts, beauty, and love. Your scientific training has emphasized the role of words, along with a strong belief in <u>reductionism</u>, hence to emphasize the possible limitations of language I shall take up the topic in several places in these Lectures. I have already discussed the fact that "style" is such a topic.

I have found that to be effective in this course, I must use mainly first hand knowledge, which implies that I break a standard taboo and talk about myself in the first person, instead of the traditional impersonal way of science. You must forgive me in this matter, as there seems to be no other approach that will be as effective. If I do not use direct experience then the Lec-

tures will probably sound to you like merely pious words and have little impact on your minds, and it is your minds I must change if I am to be effective.

This talking about first person experiences will give a flavor of "bragging", though I include a number of my serious errors to partially balance things. Vicarious learning from the experiences of others saves making errors yourself, but I regard the study of successes as being basically more important than the study of failures. As I will several times say, there are so many ways of being wrong and so few of being right, that studying successes is more efficient, and furthermore when your turn comes you will know how to succeed rather than how to fail!

I am, as it were, only a coach. I cannot run the mile for you; at best I can discuss styles and criticize yours. You know that you must run the mile if the athletics course is to be of benefit to you - hence you must think carefully about what you hear in these Lectures if they are to be effective in changing you - which must obviously be the purpose of any course. Again, you will get out of this only as much as you put in, and if you put in little effort beyond sitting in the class, then it is simply a waste of your time. Outside of class you must also mull things over, compare what I say with your own experiences, talk with others, and make some of the points part of your way of doing things.

Since the subject matter is "style, I will use the comparison with teaching painting. Having learned the fundamentals of painting, you then study under a master you accept as being a great painter; but you know you must forge your own style out of the elements of various earlier painters plus your native abilities. You must also adapt your style to fit the future, since merely copying the past will not be enough if you aspire to future greatness - a matter that I assume, and will talk about often in the Lectures. I will show you my style as best I can, but, again, you must take those elements of it that seem to fit you, and you must finally create your own style. Either you will be a leader, or a follower, and my goal is for you to be a leader. You cannot adopt every trait I discuss in what I have observed in myself and others; you must select and adapt, and make them your own if the Lectures are to be effective.

Even more difficult than what to select is the fact that what is a successful style in one age may not be appropriate to the next age! My predecessors at Bell Telephone Laboratories used one style; four of us who came in all at about the same time, and had about the same chronological age, found our own styles and as a result we rather completely transformed the overall style of the Mathematics Department, as well as many parts of the whole Laboratories. We privately called ourselves "The four young Turks", and many years later I found that top management had called us the same!

I return to the topic of education. You all recognize there is a significant difference between education and training.

Education is what, when, and why to do things,

Training is how to do it.

Either one without the other is not of much use. You need to know both what to do and how to do it. I have already compared mental and physical training and said that to a great extent in both you get out of it what you put into it - all that the coach can do is suggest styles and criticize a bit now and then. Because of the usual size of these classes there can be little direct criticism of your thinking by me, and you will simply have to do it internally and between yourselves in conversations, and apply the things I say to your own experiences. You might think that education should precede training, but the kind of educating I am trying to do must be based on a lot of your past experiences and technical knowledge. Hence this inversion of what might seem to be reasonable. In a real sense I am engaged in "meta-education", the topic of the course is education itself and hence our discussions must rise above it - "meta-education", just as metaphysics was supposed to be above physics in Aristotle's time, (actually "follow", "transcend" is the translation of "meta").

The Lectures are aimed at your future, and we must examine what is likely to be the state of technology (Science and Engineering) at the time of your greatest contributions. It is well known that since about Isaac Newton's time (1642-1727) knowledge of the type we are concerned with has about doubled every 17 years. First, this may be measured by the books published, (a classic observation is that libraries must double their holdings every 17 years if they are to maintain their relative position). Second, when I went to Bell Telephone Laboratories in 1946 they were trying to decrease the size of the staff from the war size down to about 5,500. Yet during the 30 years I was there I observed a fairly steady doubling of the employees every 17 years, regardless of the administration having hiring freezes now and then, and such things. Third, the growth of the number of scientists generally has similarly been exponential, and it is said that currently almost 90% of the scientists who ever lived are now alive! It is hard to believe that in your future there will be a dramatic decrease in these expected rates of growth, hence you face, even more than I did, the constant need to learn new things.

Here I make a digression to illustrate what is often called "back of the envelop calculations". I have frequently observed that great scientists and engineers do this much more often than "the run of the mill" people, hence it requires illustration. I will take the above two statements, knowledge doubles every 17 years, and 90% of the scientists who ever lived are now alive, and ask to what extent these are compatible. The model of the growth of knowledge and the growth of scientists assumed are both exponential, with the growth of knowledge being proportional to the number of scientists alive. We begin by assuming that the number scientists at any time t is

$$y(t) = a \exp\{bt\}$$

and that the amount of knowledge produced annually has a constant k of proportionality to the number of scientists alive. Assuming that we begin at minus infinity in time (the error is small and you can adjust it to Newton's time if you wish). We have the formula

$$1/2 = INT[-\infty, t - 17; ka \exp\{bt\}dt] / INT[-\infty, t; ka \exp\{bt\}dt]$$

$$= (ka/b)\exp\{b(t - 17)\}/(ka/b)\exp\{bt\} = \exp\{-17b\} = 1/2$$

hence we know b. Now to the other statement. If we allow the lifetime of a scientist to be 55 years (it seems likely that the statement meant living and not practicing, but excluding childhood) then we have

$$INT[t - 55, t; a \exp\{bt\}dt] / INT[-\infty, t; a \exp\{bt\}dt]$$

$$= [\exp\{bt\} - \exp\{bt - b(55)\}]/\exp\{bt\} = 1 - \exp\{-55b\}$$

$$= 1 - (1/2)^{55/17} = 1 - 0.106... = 0.894...$$

which is very close to 90%.

Typically the first back of the envelop calculations use, as we did, definite numbers where one has a feel of things, and then we repeat the calculations with parameters so you can adjust things to fit the data better and understand the general case. Let the doubling period be D, and the lifetime of a scientist be L. The first equation now becomes

$$1/2 = \exp\{bD\}$$

and the second becomes

$$9/10 = 1 - \exp\{bL\} = 1 - (1/2)^{L/D}$$

$$(1/2)^{L/D} = 1/10$$

$$L/D = \log 10 / \log 2 = 1/0.30103 = 3.3219...$$

and with d =17 years we have 17x3.3219 = 56.47... years for the lifetime of a scientist, which is close to the 55 we assumed. We can play with ratio of L/D until we find a slightly closer fit to the data (which was approximate, though I believe more in the 17 years for doubling than I do in the 90%). Back of the envelop computing suggests that the two remarks are reasonably compatible. Notice that the relationship applies for all time so long as the assumed simple relationships hold.

The reason that back of the envelop calculations are widely used by great scientists is clearly revealed - you get a good feeling for the truth or falsity of what was claimed, as well as realize what factors you were inclined not to think about, such as exactly what was meant by the lifetime of a scientist. Having

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done the calculation you are much more likely to retain the results in your mind.   Furthermore, such calculations keep the ability to model situations fresh and ready for more important applications as they arise.   Thus I recommend that when you hear quantitative remarks such as the above you turn to a quick modeling to see if you believe what is being said, especially when given in the public media like the press and TV.   Very often you find that what is being said is nonsense, either that no definite statement is made that you can model, or if you can set up the model then the results of the model do not agree with what was said.   I found it very valuable at the physics table I used to eat with; I sometimes cleared up misconceptions at the time they were being formed, thus advancing matters significantly.

Added to the problem of the growth of new knowledge is the obsolescence of old knowledge.   It is claimed by many that the half life of the technical knowledge you just learned in school is about 15 years - that in 15 years half of it will be obsolete (either we have gone in other directions or have replaced it with new material).   For example, having taught myself a bit about vacuum tubes (because at Bell Telephone Laboratories they were at that time obviously important) I soon found myself helping, in the form of computing, the development of transistors - which obsoleted my just learned knowledge!

To bring the meaning of this doubling down to your own life, suppose that you have a child when you are x years old.   That child will face, when it is in college, about y times the amount you faced.

y factor of increase	x years
2	17
3	27
4	34
5	39
6	44
7	48
8	51

This doubling is not just in theorems of mathematics and technical results, but in musical recordings of Beethoven's Ninth, of where to go skiing, of TV programs to watch or not to watch. If you were at times awed by the mass of knowledge you faced when you went to college, or even now, think of your children's troubles when they are there!   The technical knowledge involved in your life will quadruple in 34 years, and many of you will at that time be near the high point of your career.   Pick your estimated years to retirement and then look in the left hand column for the probable factor of increase over the present current knowledge when you finally quit!

What is my answer to this dilemma?   One answer is that you must concentrate on fundamentals, at least what you think at the time are fundamentals, and also develop the ability to learn new

fields of knowledge when they arise so that you will not be left behind, as so many good engineers are in the long run. In the position I found myself in at the Laboratories, where I was the only one locally who seemed (at least to me) to have a firm grasp on computing, I was forced to learn numerical analysis, computers, pretty much all of the physical sciences at least enough to cope with the many different computing problems that arose and whose solution could benefit the Labs, as well as a lot of the social and some the biological sciences. Thus I am a veteran of learning enough to get along without at the same time devoting all my effort to learning new topics and thereby not contributing my share to the total effort of the organization. The early days of learning had to be done while I was developing and running a computing center. You will face similar problems in your career as it progress, and, at times, face problems that seem to overwhelm you.

How are you to recognize "fundamentals"? One test is that they have lasted a long time. Another test is that from the fundamentals all the rest of the field can be derived by using the standard methods in the field.

I need to discuss science vs. engineering. Put glibly:

In science if you know what you are doing you should not be doing it.

In engineering if you do not know what you are doing you should not be doing it.

Of course, you seldom, if ever, see either pure state. All of engineering involves some creativity to cover the parts that are not known, and almost all of science includes some practical engineering to translate the abstractions into practice. Much of present science rests on engineering tools, and as time goes on, engineering seems to involve more and more of the science part. Many of the large scientific projects involve very serious engineering problems - the two fields are growing together! Among other reasons for this situation is almost surely that we are going forward at an accelerated pace, and now there is not time to allow us the leisure that comes from separating the two fields. Furthermore, both the science and the engineering that you will need for your future will more and more often be that which was created after you left school. Sorry! But you will simply have to actively master on your own the many new emerging fields as they arise, without having the luxury of being passively taught.

It should be noted that engineering is not just applied science, which is a distinct third field, (though it is not often recognized as such), that lies between science and engineering.

I read somewhere that there are 76 different methods of predicting the future - but that very number suggests that there is no reliable method that is widely accepted. The most trivial method is to predict that tomorrow will be exactly the same as

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today - which at times is a good bet. The next level of sophistication is to use the current rates of change and to suppose that they will stay the same - linear prediction in the variable used. Which variable you use can, of course, strongly affect the prediction made! Both methods are not much good for long term predictions, however.

History is often used as a long term guide; some people believe that history repeats itself and others believe exactly the opposite! It is obvious that:

**The past was once the future and
the future will become the past.**

In any case I will often use history as a background for the extrapolations I make. I believe that the best predictions are based on understanding the fundamental forces involved, and this is what I depend on mainly. Often it is not physical limitations that control but rather it is human made laws, habits, and organizational rules, regulations, personal egos, and inertia, that dominate the evolution to the future. You have not been trained along these lines as much as I believe you should have been, and hence I will have to be careful to include them whenever the topics arise.

There is a saying, "Short term predictions are always optimistic and long terms predictions are always pessimistic". The reason, so it is claimed, that the second part is true is that for most people the geometric growth due to the compounding of knowledge is hard to grasp. For example for money a mere 6% annual growth doubles the money in about 12 years! In 48 years the growth is a factor of 16. An example of the truth of this claim that most long term predictions are low is the growth of the computer field in speed, in density of components, in drop in price, etc. as well as the spread of computers into the many corners of life. But the field of Artificial Intelligence (AI) provides a very good counter example. Almost all the leaders in the field made long term predictions that have almost never come true, and are not likely to do so within your lifetime, though many will in the fullness of time.

I shall use history as a guide many times in spite of the fact Henry Ford, Sr. said, "History is Bunk." Probably Ford's points were:

1. History is seldom reported at all accurately, and I have found that no two reports of what happened at Los Alamos during WWII seem to agree.

2. Due to the pace of progress the future is rather disconnected from the past; the presence of the modern computer is an example of the great differences that have arisen.

Reading some historians you get the impression that the past was determined by big trends, but you also have the feeling that the future has great possibilities. You can handle this apparent

contradiction in at least four way:

1. You can simply ignore it.

2. You can admit it.

3. You can decide that the past was a lot less determined than historians usually indicate and that individual choices can make large differences at times. Alexander the Great, Napoleon, and Hitler had great effects on the physical side of life, while Pythagoras, Plato, Aristotle, Newton, Maxwell, and Einstein are examples on the mental side.

4. You can decide that the future is less open ended than you would like to believe, and that there is really less choice than there appears to be.

It is probable that the future will be more limited by the slow evolution of the human animal and the corresponding human laws, social institution, and organizations than it will be by the rapid evolution of technology.

In spite of the difficulty of predicting the future and that:

Unforeseen technological inventions can completely upset the most careful predictions,

you must try to foresee the future that you will face. To illustrate the importance of this point of trying to foresee the future I often use a standard story.

It is well known that the drunken sailor who staggers to the left or right with n independent random steps will, on the average, end up about $\sqrt{n}$ steps from the origin. But if there is a pretty girl in one direction, then his steps will tend to go in that direction and he will go a distance proportional to n. In a lifetime of many, many independent choices, small and large, a career with a vision will get you a distance proportional to n, while no vision will get you only the distance $\sqrt{n}$. In a sense, the main difference between those who go far and those who do not is that some people have a vision and the others do not and therefore can only react to the current events as they happen.

One of the main tasks of this course is to start you on the path of creating in some detail <u>your vision of your future</u>. If I fail in that I fail in the whole course. You will probably object that if you try to get a vision now it is likely to be wrong - and my reply is that from observation I have seen that the accuracy of the vision matters less than you might suppose, that getting anywhere is better than drifting, that there are potentially many paths to greatness for you, and just which path you go on, <u>so long as it takes you to greatness</u>, is none of my business. You must, as in the case of forging your personal style, find your vision of your future career, and then follow it as best you can.

No vision, not much of a future.

To what extent history does or does not repeat itself is a moot question. But it is one of the few guides you have, hence history will often play a large role in my discussions - I am trying to provide you with some perspective as a possible guide to create your vision of your future. The other main tool I have used is an active imagination in trying to see what will happen. For many years I devoted about 10% of my time (Friday afternoons) to trying to understand what would happen in the future of computing, both as a scientific tool and as shaper of the social world of work and play. In forming your plan for your future you need to distinguish three different questions:

What is possible?
What is likely to happen?
What is desirable to have happen?

In a sense the first is Science - what is possible. The second in Engineering - what are the human factors that chose the one future that does happen from the ensemble of all possible futures. The third, is ethics, morals, or what ever other word you wish to apply to value judgments. It is important to examine all three questions, and in so far as the second differs from the third, you will probably have an idea of how to alter things to make the more desirable future occur, rather than let the inevitable happen and suffer the consequences. Again, you can see why having a vision is what tends to separate the leaders from the followers.

The standard process of organizing knowledge by departments, and subdepartments, and further breaking it up into separate courses, tends to conceal the homogeneity of knowledge, and at the same time to omit much that falls between the courses. The optimization of the individual courses in turn means that a lot of important things in Engineering practice are skipped since they do not appear to be essential to any one course. One of the functions of these Lectures is to mention and illustrate many of these missed topics that are important in the practice of Science and Engineering. Another goal of the course is to show the essential unity of all knowledge rather than the fragments that appear as the individual topics are taught. In your future anything and everything you know might be useful, but if you believe that the problem is in one area you are not apt to use information that is relevant but which occurred in another course.

The course will center around computers. It is not merely that I spent much of my career in Computer Science and Engineering, rather it seems to me that computers will dominate your technical lives. I will repeat a number of times in the Lectures the following facts: Computers when compared to Humans have the advantages:

Economics - far cheaper, and getting more so
Speed - far, far faster

Accuracy -   far more accurate (precise)
Reliability -   far ahead (many have error correction built
                 into them)
Rapidity of control - many current airplanes are unstable
        and require rapid computer control to make them
        practical
Freedom from boredom - an overwhelming advantage
Bandwidth in and out - again overwhelming
Ease of retraining - change programs, not unlearn and then
        learn the new thing consuming hours and hours of
        human time and effort.
Hostile environments - outer space, underwater, high
        radiation fields, warfare, manufacturing situations
        that are unhealthful, etc.
Personnel problems - they tend to dominate management of
        humans but not of machines; with machines there are no
        pensions, personal squabbles, unions, personal leave,
        egos, deaths of relatives, recreation, etc.

   I need not list the advantages of humans over computers -
almost every one of you has already objected to this list and has
in your mind started to cite the advantages on the other side.

   Lastly, in a sense, this is a religious course - I am
preaching the message that, with apparently only one life to live
on this earth, you ought to try to make significant contributions
to humanity rather than just get along through life comfortably -
that the life of trying to achieve excellence in some area is in
itself a worthy goal for your life.   It has often been observed
that the true gain is in the struggle and not in the achievement
- that a life without a struggle on your part to make yourself
excellent is hardly a life worth living.   This, it must be ob-
served, is an opinion and not a fact, but it is based on observ-
ing many people's lives and speculating on their total happiness
rather than the moment to moment pleasures they enjoyed.   Again,
this opinion of their happiness must be my own interpretation as
no one can know another's life.   Many reports by people who have
written about the "good life" agree with the above opinion.
Notice that I leave it to you to pick your goals of excellence,
but claim only that a life without such a goal is not really
living but it is merely existing - in my opinion.   In ancient
Greece Socrates (469-399) said,

The unexamined life is not worth living.